The effect of flow parameters on mixing degree of a three dimensional rhombus micromixer with obstacles in the middle of the mixing channel using oscillatory inlet velocities

Sima Baheri Islami*, Salman Ahmadi

Faculty of Mechanical Engineering, University of Tabriz, Tabriz, Iran

ABSTRACT: The previous studies of authors on passive micromixers indicated that the micromixers dividing the flow to several layers, such as rhombus micromixers and micromixers with obstacles in the middle of the mixing channel, have higher mixing degree than other types. Also, using of oscillatory inlet velocities is an active method to enhance the mixing efficiency of micromixers. Therefore, in this study these two passive and active methods have been combined and a rhombus micromixer with obstacles in the middle of the mixing channel has been studied. Sinusoidal oscillatory velocities in two inlets have been applied with a phase difference of 180 degrees. The governing equations have been solved numerically using the finite volume method. The effect of Reynolds number, Strouhal number and amplitude on mixing degree has been investigated. Results show that degree of mixing decreases with Reynolds number at almost all Strouhal numbers. There is an optimum Strouhal number for each Reynolds number. Also, there is optimum amplitude and this optimum value decreases as Reynolds number increases.

KEYWORDS: mixing degree; oscillatory inlet velocity; passive micromixer; Strouhal number

INTRODUCTION
Microfluidic systems refer to the systems with characteristic length scales of micrometer order. The microfluids applications are extending these days such as diagnostics, lab-on-a-chip (LOC), drug delivery, sample preparation and analysis. Mixing is one of the most important problems in the biological and chemical processes which happens in microfluidic systems such as drug delivery, enzyme reactions, protein folding and lab-on-chips for complex chemical reactions.

The transport phenomena are controlled by viscous forces rather than inertia in microfluidic systems. The flow is laminar in most of these systems so it is difficult to get a uniform and rapid mixing. In these situations micromixers can be used to enhance the mixing. In general, micromixers can be classified into passive and active micromixers. Passive micromixers do not require external energy and fluids are mixed by using various arrangements of microchannels[1-3]. Various active micromixers use an external field for the mixing process such as pressure, temperature, electrohydrodynamics, magnetohydrodynamics and acoustics.

Using of oscillatory velocities with phase difference is one of the active methods to enhance the mixing. Voper et al. [4] firstly presented this method. They outlined a design for a device utilizing this approach, presented a fabricated micromixer and demonstrated its ability to move fluid using a thermally actuated bubble-pump. They enhanced mixing by perturbing a main flow in a chalet using three sets of secondary flow channels. Bottausci et al. [5] designed a micromixer which consisted of a main rectangular channel and three cross-stream secondary channels. Their results showed that mixing was substantially improved with multiple side channels with oscillatory flows, whose frequencies were increasing downstream. Nguyen et al. [6] investigated a micromixer based on combined hydrodynamic focusing and time-interleaved segmentation, theoretically and experimentally. They indicated that hydrodynamic focusing reduced the transversal mixing path while time-interleaved sequential segmentation shortened the axial mixing path. They also presented a time-dependent two-dimensional analytical model for the mixing concept. According to their results the concentration profile along the mixing channel agreed qualitatively well with the analytical model. Nguyen et al. [7] introduced a simple theoretical solution for oscillatory entrance flow in microchannels and found out that the Peclet number was an important parameter in mixing. Goullet et al. [8] showed that mixing was affected by both the geometry of confluence of two inlet microchannels and the inclusion of features in the channels, which induced secondary flow. The results indicated that pulsed flow technique was more effective at mixing than the secondary flow induced by the channel geometry features, so combining both methods leads to even better mixing. Kim et al. [9] analyzed the mixing behavior of an active micromixer
equipped successively with a circular cylinder and an oscillating stirrer using the lattice Boltzmann method. Results indicated that the composite active mixer with an established circular cylinder upstream of the stirrer improved the mixing efficiency by 27% over an active mixer with only a stirrer. Ma et al. [10] examined the performance of an unsteady microfluidic T-form mixer driven by pressure disturbances through both numerical simulation and experimentation. They investigated the mixing efficiency by optimization of the disturbance frequency, which was represented by the Strouhal number for the fixed base flow rate. Chen and Cho [11] numerically analyzed the respective effects on the mixing efficiency of the geometric amplitude of the wavy surface, the length of the wavy-wall section, and the Strouhal number of the periodic velocity perturbations with phase difference of 180°. The results revealed that the mixing performance was improved by increasing the geometric wave amplitude or length of the wavy-wall section and by applying a Strouhal number in the range 0.33–0.67. Wang et al. [12] numerically investigated the performance of a magnetic particle driven micromixer. This micromixer had taken the advantages of mixing enhancements induced by alternating actuation of magnetic particles suspended in the fluid. They indicated that the maximum efficiency is obtained at a relatively high operating frequency for large magnetic actuation forces. Miranda et al. [13] numerically studied the mixing in microsystems, combining alternate flow with obstacles. Simulations showed that the layers of high and low solute concentrations, created by the alternate flow, were split into smaller chunks of fluid by obstacles in the mixing channel. In addition the results indicated that mixing increased with the increase of Strouhal number and the number of obstacles. Sun and Sie [14] investigated the active mixing in diverging microchannels experimentally. They reported that the presence of both flow instability and the resulting tendril structures, which provide extra stretching and folding of the interface, are necessary to achieve excellent mixing in the diverging microchannels. Lim et al. [15] presented a new approach to enhance mixing in T-type micromixers by introducing a constriction in the microchannel under periodic electro-osmotic flow. Two sinusoidal AC electric fields with 180° phase difference and similar DC bias were applied at the two inlets. They reported that the crescent-shaped layers of fluids which were formed due to the constriction introduced at the junction, increased tremendously the contact surface area between the two streams of fluid and thus enhanced significantly the mixing efficiency.

Authors compared the mixing degree of nine different micromixer geometries in the mixing of two fluid fluids under sinusoidal oscillatory velocities with a phase difference of 180 degrees relative to each other [16]. Simple T-shaped micromixer, micromixer with rectangular or parallelogram ribs on the walls of the mixing channel, T-shaped micromixer with two additional parallel or perpendicular inlet channels, micromixer with circular or triangular barriers in the middle of the mixing channel, rhombus micromixer with thick or thin edges, were investigated. Results indicated that the micromixers, which divide the flow to several layers such as rhombus micromixers and micromixers with obstacles in the middle of the mixing channel, have higher mixing degree than others. The geometry of current study is based on the results of author’s previous study [16]. In current study, a three dimensional rhombus micromixer with obstacles in the middle of the mixing channel has been studied. Two entrance flows are oscillatory with phase difference. Also, the effect of Reynolds number, Strouhal number and the amplitude of oscillations on the mixing degree has been investigated.

GOVERNING EQUATIONS

The governing equations of three dimensional, unsteady incompressible Newtonian fluid flows are continuity, momentum and mass fraction conservation:

\[ \nabla \cdot \vec{V} = 0 \]  \hspace{1cm} (1)
\[
\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla) \vec{V} = -\frac{1}{\rho} \nabla P + \nu \nabla^2 \vec{V}
\] (2)

\[
\frac{\partial Y_i}{\partial t} + (\vec{V} \cdot \nabla) Y_i = D \nabla^2 Y_i
\] (3)

where, \( \vec{V} \) is the velocity vector, \( \rho, P \) and \( \nu \) are the density, pressure and kinematic viscosity. \( Y_i \) is the mass fraction of \( i \)-th component of mixture and \( D \) is the mass diffusion coefficient. For two component mixtures, the mass fraction of one component \((Y_1)\) is obtained by solving equation3 and the mass fraction of other component is obtained by \( Y_2 = 1 - Y_1 \). One important parameter in mixers is degree of mixing, which can be defined in each section of channel as [8]:

\[
DM = 1 - \frac{\sum_{j=1}^{n} \frac{(Y_{ij} - \bar{Y})^2}{u_j / u_{mean}}}{\bar{Y}}
\] (4)

In this equation \( Y_{ij} \) is the mass fraction of \( i \)-th component of mixture in \( j \)-th cell, \( u_j \) is the tangential velocity of mixture in \( j \)-th cell, \( u_{mean} \) is the mean velocity of mixture in the used section, \( \bar{Y} \) is the mass fraction of perfect mixing (which has been assumed 0.5 in this study) and \( n \) is the number of cells in the used section. The frequency and mean velocity of both inlets are equal and can be presented as non-dimensional Strouhal and Reynolds numbers:

\[
St = \frac{f d_h}{\bar{v}}
\] (5)

\[
Re = \frac{\bar{v} d_h}{\nu}
\] (6)

where \( d_h \) is the hydraulic diameter of inlet section, \( \bar{v} \) is the mean velocity and \( f \) is frequency.

GEOMETRY AND BOUNDARY CONDITIONS

The geometry of three dimensional rhombus micromixer is shown in Figure 1 (all dimensions are in mm).

A pure fluid enters from bottom inlet and the same fluid containing a tracer enters from top inlet. In this study it is assumed that the tracer do not change the physical properties of pure fluid so, the kinematic viscosity and mass diffusion coefficient of both fluids are \( \nu = 10^{-6} \) (m/s) and \( D = 10^{-10} \) (m²/s), respectively. The mass fraction of tracer at the top and bottom inlets is 1 and 0, respectively.

The inlet velocities are oscillatory and defined as:

\[
v_i(t) = \bar{v}_i (1 + a_i \sin(2\pi f_i t + \phi_i))
\] (7)

where \( i \) is 1 or 2, respectively for top or bottom inlet. \( a_i \) is the non-dimensional amplitude and \( \phi_i \) is the phase, which is \( \pi \) for top inlet and 0 for bottom inlet. For all of the walls no-slip boundary condition was applied.

Three Reynolds numbers: \( Re = 0.156, 0.78, 1.56 \), six Strouhal numbers: \( St = 0.025, 0.051, 0.102, 0.204, 0.408, 0.612 \), and six non-dimensional amplitudes: \( a = 4, 7, 14, 28, 56, 84 \) were studied in this study.

Grid independency

The grid independency was checked for all results. One example is presented in Figure 2 as the variation of degree of mixing with cells number for \( Re = 1.56 \) and \( St = 0.102 \) at 1 mm after the T-junction and \( t = 0.3 \) s.

![Fig. 2. Grid independency test, Re=1.56, St= 0.102, x= 1mm](image)

The cells number of \( N = 235260 \) was selected for this case and it is clear that more increment of cells number do not have considerable effect on \( DM \) except that increases the calculation time.

Validation of Numerical results

The numerical results of current study have been compared with the results of Ma et al. [10]. They studied the mixing of two fluids with oscillatory inlet velocities using a simple T-shape micromixer, experimentally and numerically. The mixing degree variations versus distance from junction in Figure 3a and mass fraction contours in Figure 3b are compared with ref. [10]. It can be seen that there is a good agreement between results.
The effect of Reynolds number

The effect of Reynolds number on mixing is shown in Figure 4 for various Strouhal numbers. It is clear that the degree of mixing decreases as the Reynolds number increases at almost all of the Strouhal numbers.
It can be explained by definition of the mixing time by diffusion and residential time of two fluids in a mixing channel.

\[
t_m = \frac{d_h}{L} \frac{Re \cdot Sc}{\nu}
\]

where \( L \) is the mixing channel length and \( Sc \) is the Schmidt number and is defined as:

\[
Sc = \frac{\nu}{D}
\]

It is clear that this ratio increases with increasing the Reynolds number. Therefore, the fluids have not enough time to mix by diffusion and mixing degree decreases. As the Strouhal number increases the contact area between two fluids increases, so the effect of Reynolds number becomes more considerable. But Figure 4 shows that the influence of Reynolds number begins to be reversed at Strouhal numbers higher than 0.408. At high Strouhal numbers, as Reynolds number increases, a circulation zone near the upper corner of junction creates and becomes an auxiliary means to improve mixing efficiency. Figures (5-7) show the streamlines on the mid plane of the micromixer for various Reynolds and Strouhal numbers. Corner circulation zones can be seen in these figures. There is an intention of both fluids stream towards inlets instead of mixing channel, because of oscillatory velocities (figures 6 and 7, inside rectangular boxes). The created circulation zones at high Reynolds number circulate the both streams and direct them towards the mixing channel and enhance the mixing compared to low Reynolds number. Since the circulating zone is bigger at \( Re=1.56 \) than the one of \( Re=0.78 \), the effect of Reynolds number reverses at high frequencies.

![Fig. 5. Streamlines at z=0, Re=0.156, t=10 s, (a) St=0.102 (b) St=0.204 (c) St=0.408 (d) St=0.612](image)
The effect of Strouhal number

Figure 8 shows the variation of degree of mixing with Strouhal number for three Reynolds numbers. It is clear that there is an optimum Strouhal number for each Reynolds number and this optimum decreases by increasing the Reynolds number. The reason behind this is that increasing of Reynolds number decreases the residential time of fluids in the channel, as mentioned before, and therefore, mixing decreases. Furthermore, Figure 8 shows that the reduction rate of mixing index is slower for Re=0.156 than higher Reynolds numbers.
Fig. 7. Streamlines at z=0, Re=1.56, t=10 s, (a) St=0.102 (b) St=0.204 (c) St=0.408 (d) St=0.612

Fig. 8. Variation of degree of mixing versus Strouhal number
Figure 9 shows the mass fraction contours of fluid 1. The trends in Figure 8 can be explained by using figures (5-7) and Figure 9. At low frequencies, the number of layers of two fluids is lower so, the contact area of the fluids is lower and accordingly the degree of mixing is smaller (The layers of fluid can be recognized by black and white colored fluids, which enter to the channel).

As the frequency increases, the number of fluid layers in the mixing channel increases at the moment and as a result, the contact area and mixing degree increases. But, at very high frequencies (figure 9e and 9f) before the entrance of one fluid to the mixing channel the other fluid reaches and both fluids enter the mixing channel simultaneously, which leads to the reduction of contact area and fluids layers, and consequently degree of mixing. As inlet velocities and so their momentum increases, the possibility of the penetration of one fluid into another fluid’s inlet channel instead of mixing channel raises. It is evident from Figures 5 and 6 when you compare the streamlines of Re=0.156 and Re=0.78. That is why the reduction rate of mixing index is higher for Re=0.78 than Re=0.156.

**The effect of amplitude**

The effect of non-dimensional amplitude in the mixing is illustrated in Figure 10 for two Strouhal numbers. It can be observed that there is an optimum amplitude for both Strouhal numbers and this optimum value decreases as Reynolds number increases.
Fig. 10. Variation of degree of mixing versus non-dimensional amplitude, (a) St=0.102 (b) St=0.204

For example the optimum amplitude at Re=0.78 and St=0.204 is a=16. This can be seen in Figure 11, which shows the concentration distribution in the cross section of the mixing channel at x=0.5 mm for Re=0.78 and St=0.204. The concentration distribution is symmetric relative to the channel center, for a=16; therefore, it has better mixing degree. At a=8 the black fluid tends to be inclined towards the bottom of the channel, while at a=22 and a=28 it goes to the top of the mixing channel.

Fig. 11. Mass fraction contours of fluid 1, x=0.5mm, Re=0.78, St=0.204;(a) a=8  (b) a=16  (c) a=22 (d) a=28

CONCLUSIONS
In current study, the mixing efficiency of rhombus micromixer with obstacles in the middle of the mixing channel was studied numerically. Sinusoidal oscillatory velocities used as a means to enhance the mixing of two fluids. The effect of flow parameters on mixing degree was
investigated. Results indicated that degree of mixing decreases with Reynolds number at moderate Strouhal numbers and this effect reverses at high frequencies. Also, there is an optimum Strouhal number and amplitude for each Reynolds number. The optimum value of amplitude decreases as Reynolds number increases. The creation of circulation zones near the micromixer junction at high Reynolds and Strouhal numbers can change the fluids mixing behavior.

REFERENCES


