**Numerical Simulation of Laminar Convective Heat Transfer and Pressure Drop of Water Based-Al$_2$O$_3$ Nanofluid as A Non Newtonian Fluid by Computational Fluid Dynamic (CFD)**

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**ABSTRACT:** The convective heat transfer and pressure drop of water based Al$_2$O$_3$ nanofluid in a horizontal tube subject to constant wall temperature condition is investigated by computational fluid dynamic (CFD) method. The Al$_2$O$_3$ nanofluid at five volume concentration of 0.1, 0.5, 1.0, 1.5 and 2 % are applied as a non Newtonian power law and Newtonian fluid with experimentally measured properties of density, viscosity, thermal conductivity and specific heat capacity. The power law fluid determines the heat transfer coefficient and pressure drop better than that of the Newtonian fluid. The experimentally measured viscosity is used as consistency index and the flow behavior index ($n$) is computed in various Reynolds number and nanoparticle concentrations in order to minimize the difference between the experimental and computational results. It is revealed that $n$ is a function of nanoparticle concentration and independent of nanofluid velocity and Al$_2$O$_3$ nanofluid behaves as a shear thickening fluid for $n$>1. Both the experimental and computational results show an increase in the heat transfer coefficient and pressure drop with an increase in the nanofluid concentration. By using the experimental data a correlation for the average Nusselt number estimation based on the dimensionless number (Re and Pr) and nanoparticles concentration ($\phi$) is obtained. The results of this correlation introduce a 1.162 % average absolute deviation.

**KEYWORDS:** Nanofluid; Convective Heat Transfer; Power Law Fluid; Laminar Flow; Numerical Simulation

**INTRODUCTION**

Nanofluids are prepared by dispersing the metallic and nonmetallic nanoparticles in a based fluid. The based fluid can be water, ethylene glycol, motor oil..., and the nanoparticles can be carbon nanotubes, TiO$_2$, Al$_2$O$_3$, γ-Al$_2$O$_3$,... Nanofluids have suitable properties such as: high thermal conductivity, improved heat transfer coefficient and minimal clogging in flow. These characteristics make the nanofluids to become suitable potential in heat transfer system as far as high thermal efficiency is concerned. There exist many experiments on the laminar and turbulent flow of nanofluids in tube, heat exchanger, helical coil [1-5]. Pak and Cho [6] were the first to report on the convective heat transfer of Al$_2$O$_3$ and TiO$_2$ nanoparticles dispersed in water in a horizontal tube. Xaun and Li [7] reported an enhancement in the heat transfer coefficient of 40% for Cu-water nanofluid flowing through a straight tube with a constant heat flux at the wall subject to a laminar and turbulent flow condition. Wen and Ding [8] measured the local heat transfer coefficient of Al$_2$O$_3$-water nanofluid inside a copper tube subject to a constant wall heat flux. They explained that the heat transfer coefficient increase was not just for fluid thermal conductivity improvement.

Other parameters such as particle migration and Brownian motion cause this improvement [9]. Ding et al. [10] revealed that the heat transfer behavior of multi-walled carbon nanotubes suspension at a constant wall heat flux. Williams et al. [11] found that the ratio of heat transfer rate to required pumping power for Al$_2$O$_3$-water and Zirconia-water nanofluids. They found that this ratio for nanofluids was lower than pure water because of an increase in viscosity. Chun et al. [12] performed an experimental study on the convective heat transfer of three alumina nanoparticles in transformer oil as the based fluid flowing through a double pipe heat exchanger subject to a laminar flow condition. Zamzamian et al. [13] studied the forced convective heat transfer coefficient of Al$_2$O$_3$-EG and CuO-EG nanofluids in a double pipe heat exchanger at different operating temperatures subject to the turbulent condition. Xie et al. [14] investigated the enhancement of heat transfer coefficient for ZnO, Al$_2$O$_3$, MgO, and TiO$_2$ nanofluids in a circular channel subject to a constant wall temperature.

Since, the nanofluids do not behave as a Newtonian fluid, particularly at high concentrations of the nanoparticles, in this article the heat transfer and pressure drop determinations for Al$_2$O$_3$ nanofluid are simulated at five concentrations of nanoparticle through numerical solving of the governing equations. The results are
Nomenclature

- $u$: Fluid velocity (m.s$^{-2}$)
- $h$: Heat transfer coefficient (W.m$^{-2}$.K$^{-1}$)
- $A$: Cross section area (m$^2$)
- $c_p$: Specific heat capacity (J.Kg$^{-1}$.K$^{-1}$)
- $k$: Thermal conductivity (W.m$^{-1}$.K$^{-1}$)
- $g$: Gravitational acceleration (m.s$^{-2}$)
- $h$: Heat transfer coefficient (W.m$^{-2}$.K$^{-1}$)
- $A$: Cross section area (m$^2$)
- $n$: Flow behavior index
- $T$: Temperature (K)
- $v$: Velocity (m.s$^{-1}$)
- $D$: Tube diameter (m)
- $Nu$: Nusselt number
- $Re$: Reynolds number
- $Pr$: Prandtl number
- $\rho$: Density (Kg.m$^{-3}$)
- $l$: Length of tube (m)
- $q$: Spectral irradiance, power density (W.m$^{-3}$)
- $\mu$: Viscosity (Kg.m$^{-1}$.s$^{-1}$)
- $k$: Thermal conductivity (W.m$^{-1}$.K$^{-1}$)
- $\phi$: Nanoparticles volume fraction (Vol %)
- $\tau$: Shear stress tensor
- $\gamma$: Shear rate tensor
- $\tilde{\gamma}$: Magnitude of shear rate tensor
- $K$: Consistency index
- $n$: Flow behavior index
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Subscripts

- $nf$: Nanofluid
- $bf$: Basefluid
- $m$: Mean
- $w$: Wall
- $np$: Nanoparticle

MATHEMATICAL MODELING

For flow behavior and heat transfer prediction the continuity, momentum and energy equations are represented as follows:

The continuity equation:

$$\nabla \cdot (\rho_{nf} \mathbf{v}) = 0 \tag{1}$$

The momentum equation in terms of shear stress:

$$\rho \frac{Dv}{Dt} = -\nabla p - [\nabla \cdot \tau] + \rho g \tag{2}$$

The energy equation:

$$\rho C_p \frac{DT}{Dt} = (\nabla \cdot q) - \left( \frac{\partial \ln p}{\partial \ln T} \right)$$

Here, $\frac{Dp}{Dt} = (\tau : \nabla v)$

The experimental data obtained by Heyhat et al. [15] compared with the experimental data obtained by Heyhat et al. [15].

The experimental data for Al$_2$O$_3$-nanofluid show a good agreement with the mixing theory.

$$\rho_{nf} = \rho_{np} + (1 - \phi) \rho_{bf} \tag{6}$$

The experimental specific heat capacity is covered by Equation 7.

The specific heat capacity is calculated by dividing Equation 7 by the fluid density.

$$C_{p,nf} = \frac{\phi (\rho C_p)_{np} + (1 - \phi) (\rho C_p)_{bf}}{\rho_{nf} + (1 - \phi) \rho_{bf}} \tag{7}$$

In this study the rheological behavior of nanofluid is estimated by a power-law model because the suspensions such as nanofluid must have a shear thickening behavior. The shear stress by a power-law model with $n>1$ reveals the following manner.

$$\tau = K (\dot{\gamma})^{n-1} (\dot{\gamma}) \tag{8}$$

where, $\dot{\gamma}$, $\gamma$, $K$ and $n$ are shear rate tensor, magnitude of shear rate tensor, consistency index and flow behavior index, respectively.

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simulation of steady state, fully developed flow in a horizontal pipe and an axiymmetric assumption made for solving the governing equations in cylindrical coordinates. For numeric simulation the input fluid properties (density, viscosity, thermal conductivity and specific heat capacity) are the experimental data of the $\text{Al}_2\text{O}_3$ nanofluid at different concentrations.

The first order upwind finite volume scheme is adopted for the discretization of the momentum and energy equations.

Heat transfer and pressure drop drop evaluation

The convective heat transfer coefficient and Nusselt number are calculated through the following equations:

$$\overline{h_{nf}} = \frac{C_{p_{nf}} \rho_{nf} \overline{u_m} A (T_{mo}-T_{mf})}{\pi D L (T_w - T_{nf})} = \frac{q_w^*}{(T_w - T_{nf})}$$  \hspace{1cm} (9)

where, $A$ and $T_{nf}$ are the cross section areas and mean temperature on fluid volume and $T_{mo}$ and $T_{mi}$ are the mean temperatures on pipe outlet and inlet, respectively. The average Nusselt number in pipe length can be defined by the average heat transfer coefficient ($\overline{h_{nf}}$).

$$\overline{Nu_{nf}} = \frac{\overline{h_{nf}} D}{k_{nf}}$$  \hspace{1cm} (10)

The Hagen Poiseuille laws for Newtonian and power-law fluid are presented by Equations 11 and 12, respectively. These equations are solved numerically for pressure drop calculation along the pipe.

$$\Delta p = \frac{32 \mu L u_m}{\rho^2}$$  \hspace{1cm} (11)

$$\Delta p = \frac{2^{n+2} \left( \frac{3n+1}{n} \right)^n L K u_m^n}{D^{n+1}}$$  \hspace{1cm} (12)

RESULTS AND DISCUSSION

Pure water

Pure water, as the base fluid is simulated to validate the experimental results of heat transfer and pressure drop. The experimental heat transfer coefficient is illustrated in Figure 1 at various Reynolds number within the laminar region. The simulation is carried out by applying the Newtonian model (power-law with $n=1$) because water is considered as a completely Newtonian fluid.

Figures 2 and 3 show the average Nusselt number and pressure drop for pure water versus Reynolds number. The Newtonian model for pure water is in a good agreement with the experimental data.

$\text{Al}_2\text{O}_3$ nanofluid

Heat transfer

Nanofluids are considered as solid-liquid suspensions; therefore, by an increase in shear rate they got closer to one another and cause an increase in viscosity, which is called shear thickening behavior.

Power-law model shows this behavior as $n>1$. In this study the fluid behavior index ($n$), as an adjustable parameter is estimated to minimize the difference between the experimental and the simulation data. For this purpose
the calculated Reynolds number is applied by changing the $n$ magnitude from 0.1 to 2 at each concentration of the nanoparticles. The results show that the $n$ magnitude increases by an increase in the solid concentration.

Figure 4 shows the simulated results at two concentrations of 2 and 0.5 Vol % of the nanoparticles and at two Reynolds number (high and low). For example as illustrated in Figure (4a), a minimum difference of $\text{Nu}_\text{Exp}$ (5.050) and $\text{Nu}_\text{Sim}$ (5.22) is obtained at 2 Vol % of the nanoparticles and Re=1862 with $n=1.6$. A decrease in the nanoparticles concentration decreases the $n$ ($n\to1$ or Newtonian fluid).

The Equation 13 which correlates the $n$ magnitude in terms of the nanoparticles concentration ($\phi$) is obtained through the simulation runs at all concentrations of Al$_2$O$_3$ nanofluid (0.1, 0.5, 1.0, 1.5 and 2 Vol %).

$$n = -0.0759\,\phi^2 + 0.4070\,\phi + 1.0790 \quad (13)$$

![Figure 4(a) Simulation Nu number as function of $n$ for 2 Vol % Al$_2$O$_3$ nanofluid (for Re=1862 $\text{Nu}_\text{Exp}=5.050$ and for Re=389, $\text{Nu}_\text{Exp}=4.104$)](image)

![Figure 4(b) Simulation Nu number as function of $n$ for 0.5 Vol % Al$_2$O$_3$ nanofluid (for Re=1915, $\text{Nu}_\text{Exp}=4.858$ and for Re=488, $\text{Nu}_\text{Exp}=3.961$)](image)

Figure 5 illustrates the experimental [15] and the simulated average heat transfer coefficients versus Reynolds number for five concentrations of the nanoparticles.

The absolute averages deviation of the numerical results for the power law model at the optimum $n$ magnitudes are less than the Newtonian model. The results of the Newtonian model are approximately close to the results of power-law model at low concentrations of nanoparticles, ($\phi=0.1$).

Figure 6 shows the experimental [15] and the simulated average Nusselt number at different Reynolds number for five concentrations of the nanoparticles. Both the experimental and the simulation results show an increase in the average Nusselt number which is not observed in pure water.

**Pressure drop**

In general, the applied nanofluid improves the heat transfer coefficient while increasing the nanoparticles concentration in the base fluid has an undesirable effect on the pressure drop in pipe and increases the fluid pumping power; therefore, it is necessary to study the viscose pressure loss of nanofluid.

Figure 7 shows the experimental pressure drop of nanofluid [15] and the values which are simulated by the power-law and Newtonian models.

As seen, at high concentration of Al$_2$O$_3$ nanoparticles the Newtonian model fails to predict the pressure drop while the power-law model reveals a good prediction for the experimental results. At low concentration ($\phi=0.1$ Vol %) the nanofluid behavior prediction approaches to the Newtonian model results and both the models have good estimations.

**Nanofluid heat transfer correlation**

The experimental results of nanofluid heat transfer obtained by Heyhat et al. [15] can be applied for a particular correlation for the average Nusselt number as a function of Reynolds, Prandtl and nanoparticles concentration ($\phi$).

Equation 14 shows this correlation for the laminar convective heat transfer of Al$_2$O$_3$ nanofluid. A comparison between this correlation and the experimental Nusselt number [15] is illustrated in Figure 8.

As seen, the results of this correlation deviate at 5 % in maximum; while, the absolute average deviation is recorded as 1.162 %.

$$\overline{\text{Nu}}_{\text{nf}} = 0.8969 \, \text{Re}^{0.3710} \, \text{Pr}^{0.0085} \, \phi^{-0.1399} \quad (14)$$
Fig. 5. Comparison of experimental average heat transfer coefficient [15] and the simulated results by power-law and the Newtonian model. (a) \( \phi = 2.0 \) Vol %, AAD % for Power law = 3.24%. (b) \( \phi = 1.5 \) Vol %, AAD % for Power law = 3.96%. (c) \( \phi = 1.0 \) Vol %, AAD % for Power law = 5.81%. (d) \( \phi = 0.5 \) Vol %, AAD % for Power law = 5.09%. (e) \( \phi = 0.1 \) Vol %, AAD % for Power law = 3.97%.
Fig. 6. Comparison of experimental average Nusselt number [15] and the simulated results by power-law and the Newtonian model (a) φ = 2.0 Vol % (b) φ = 1.5 Vol % (c) φ = 1.0 Vol % (d) φ = 0.5 Vol % (e) φ = 0.1 Vol %
Fig. 7. Comparison of experimental pressure drop [15] and the numerical simulation results by power-law and the Newtonian model (a) φ=2.0 Vol % (b) φ=1.5 Vol % (c) φ=1.0 Vol % (d) φ=0.5 Vol % (e) φ=0.1 Vol %
CONCLUSION

In this article the laminar convective heat transfer and pressure drop for a water based $\text{Al}_2\text{O}_3$ nanofluid in a horizontal tube subject to a constant wall temperature is simulated by the power-law and Newtonian models numerically.

The power-law model shows better agreement with respect to the experimental data than that of the Newtonian model.

REFERENCES


