

Numerical simulation of nanofluid flow over diamond-shaped elements in tandem in laminar and turbulent flow

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ABSTRACT: In this paper, the Al₂O₃-water nanofluid flow in laminar and turbulent flows inside tubes fitted with diamond-shaped turbulators is numerically modeled. The nanofluid flow is modeled by employing a two-phase mixture method and applying the constant heat flux boundary condition at tube walls. In the results, the effects of different parameters such as the geometry of turbulators, volume fraction and diameter of nanoparticles, etc. on the flow field in the tubes have been investigated. The obtained results indicate that, with the reduction of tail length ratio (*TR*) and increase of vertex angle of turbulators (θ), the heat transfer coefficient as well as the wall shear stress increase. Similarly, with the reduction of *TR* and increase of θ , the amount of secondary flows, vortices and the turbulent kinetic energy increase. Moreover, the increase in the volume fraction of nanoparticles and the reduction of nanoparticles diameter lead to the increase of the heat transfer coefficient and wall shear stress.

KEYWORDS: Nanofluid; Diamond-shaped turbulators; CFD; Turbulent flow; Mixture model

INTRODUCTION

Increasing the amount of heat transfer in different engineering applications such as the cooling of electronic components, cooling and boiling of liquids, design and operation of heat exchangers, etc. is highly important. The methods of heat transfer improvement can be generally divided into two groups of active and passive techniques. The passive heat transfer enhancement method, due to its simplicity of use and lower cost, has greatly attracted the attention of industrialists and researchers in recent years [1-6].

A very important and useful way of passively increasing the amount of heat transfer is the use of turbulators in flow passage to induce and promote the flow turbulence and to increase the local Reynolds number. In this regard, numerous investigations have been conducted in recent years both numerically and experimentally. Eiamsa-ard and Promvonge [1] experimentally studied the flow of air in circular tubes fitted with diamond-shaped turbulators. They explored the effects of the tail length ratio and also the vertex angle of the diamond-shaped turbulators on the heat transfer coefficient and the friction factor of the fluid. Ayhan et al. [2] experimentally and numerically investigated the increase in the amount of heat transfer in circular tubes containing truncated hollow cone turbulators. Durmus [3] evaluated the improvement of heat transfer in 4 different cut out conical turbulators.

He compared the heat transfer coefficient, pressure drop and exergy values obtained with and without the use of turbulators. Eiamsa-ard and Promvonge [4] investigated the improvement of heat transfer in tubes fitted with V-shaped turbulators. They demonstrated that these turbulators can increase the amount of heat transfer by 270% relative to flows in ordinary circular tubes. Promvonge [5] studied the effects of different diameters and arrangements of conical turbulators in fluid flows on the heat transfer and friction coefficient values of tubes. Akansu [6] numerically investigated the characteristics of fluid flow in tubes containing porous rings and demonstrated that the highest amount of heat transfer is obtained at the ratio of $H/d = 2$.

Another technique for enhancing the heat transfer in tubes is the use of a nanofluid instead of base fluid. A nanofluid refers to a compound in which solid, and mostly metallic, particles at nano sizes (usually less than 100 nm) are added to an ordinary fluid and help increase the value of the mixture conductivity and thus improve the amount of heat transfer in that fluid. Due to a considerable improvement of heat transfer and a negligible pressure drop achieved by nanofluids, relative to ordinary fluids, the use of nanofluids has become very commonplace in recent years [7-10]. Also in recent years, the combined use of nanofluids in conjunction with other heat transfer improvement mechanisms such as twisted tapes and coiled wires has been on the rise [11-12]. Nevertheless, based on our information, no research has been conducted so far on the subject of nanofluid flow in tubes containing diamond-

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Nomenclature		Greek Symbols	
a	Acceleration (m s ⁻²)	β	Volumetric expansion coefficient (K ⁻¹)
C _p	Specific heat (J kg ⁻¹ K ⁻¹)	δ	Distance between particles (m)
C	Constant in Eq. (14)	Φ	Nanoparticles volume fraction
d _p	Diameter of nanoparticles (m)	θ	Semi vertex angle of turbulators
g	Gravitational acceleration (m s ⁻²)	λ _r	Mean free path of water molecular (m)
h	Local heat transfer coefficient (W/m ² K)	μ	Dynamic viscosity (N s m ⁻²)
k	Thermal conductivity (W m ⁻¹ K ⁻¹)	ν	Kinematic viscosity (m ² s ⁻¹)
k _B	Boltzmann constant (=1.3807 × 10 ⁻²³ J K ⁻¹)	Subscripts	
L	Length of tubes (m)	BF	Base fluid
L _h	Head length of turbulator (m)	dr	Drift
L _t	Tain length of turbulator (m)	f	Fluid
Pr	Prandtl number (=α _m /ν _m)	i	Inlet conditions
q	Heat flux (W m ⁻²)	k	Indices
Re	Reynolds number (=VD _h /ν _m)	m	Mixture
T	Temperature (K)	p	Nanoparticle phase
TR	Tail length ratio (=L _t /L _h)	w	Wall
V	Velocity (m s ⁻¹)	α	Thermal diffusivity (=k/ρc _p)

shaped turbulators. In this paper, the effects of simultaneously applying two heat transfer enhancement mechanisms, namely, the use of nanofluids and also diamond-shaped turbulators in laminar and turbulent flows have been numerically investigated.

MATHEMATICAL MODELING

Geometry

The geometries evaluated in this paper include different circular tubes fitted with diamond-shaped turbulators. Figure 1 shows the schematic of these tubes. The effects of different geometrical parameters such as *TR* and *θ* on the behavior of nanofluid flow will be investigated in this paper. The results will be obtained and displayed for 3 different *TR* and 3 different *θ* values.

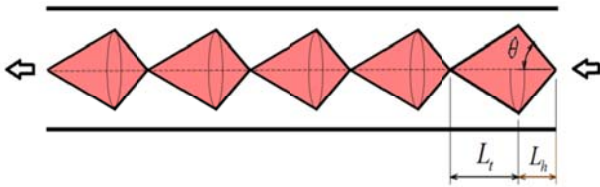


Fig.1. Detailed definition of diamond-shaped turbulators in tube

Governing equations

In the present study, numerical simulation of nanofluid flow is performed using mixture model which is a single fluid two phase approach. This method investigates equilibrium over spatial length scales. In this method each phase has its own velocity field, and in a given control volume there is a certain fraction of base fluid and nanoparticles.

Instead of utilizing the governing equations of each phase separately, it solves the continuity, momentum and energy equations for the mixture, and the volume fraction

equation for nanoparticles. The equations for the steady state conditions and turbulent mean flow are as follows:

Continuity:

$$\nabla \cdot (\rho_m V_m) = 0 \quad (1)$$

Momentum:

$$\begin{aligned} \nabla \cdot (\rho_m V_m V_m) &= -\nabla p_m + \nabla \cdot (\tau - \tau_t) + \\ \nabla \cdot \left(\sum_{k=1}^n \phi_k \rho_k V_{dr,k} V_{dr,k} \right) &+ \rho_m g \end{aligned} \quad (2)$$

Energy:

$$\begin{aligned} \nabla \cdot \left(\sum_{k=1}^n \rho_k C_{pk} \phi_k V_k T \right) &= \nabla \cdot (k_m \nabla T - \\ \rho_m C_{pm} \overline{v} t) \end{aligned} \quad (3)$$

Volume fraction:

$$\nabla \cdot (\phi_P \rho_P V_m) = -\nabla \cdot (\phi_P \rho_P V_{dr,P}) \quad (4)$$

Where V_m is the mass average velocity:

$$V_m = \frac{\sum_{k=1}^n \phi_k \rho_k V_k}{\rho_m} \quad (5)$$

In Equation 2, $V_{dr,k}$ is the drift velocity for nanoparticles:

$$V_{dr,k} = V_k - V_m \quad (6)$$

The slip velocity (relative velocity) is calculated as the velocity of nanoparticles relative to the velocity of base fluid:

$$V_{pf} = V_p - V_f \quad (7)$$

The relation between drift velocity and relative velocity is as follows:

$$V_{dr,p} = V_{pf} - \sum_{k=1}^n \frac{\phi_k \rho_k}{\rho_m} V_{fk} \quad (8)$$

The relative velocity and drag function are calculated using Manninen et al. [13] and Schiller and Naumann [14] relations respectively, as follows:

$$V_{pf} = \frac{\rho_p d_p^2}{18 \mu_f f_{drag}} \frac{(\rho_p - \rho_m)}{\rho_p} a \quad (9)$$

$$f_{drag} = \begin{cases} 1 + 0.15 \text{Re}_p^{0.687} & \text{for } \text{Re}_p \leq 1000 \\ 0.0183 \text{Re}_p & \text{for } \text{Re}_p > 1000 \end{cases} \quad (10)$$

The acceleration (a) in Eq. (9) is

$$a = g - (V_m \cdot \nabla) V_m \quad (11)$$

The shear stress relation is given by:

$$\tau = \mu_m \nabla V_m \quad (12)$$

$$\tau_t = \sum_{k=1}^n \rho_k \phi_k \overline{v_k v_k v_k} \quad (13)$$

Turbulence modeling

In this paper, as suggested by Namburu et al. [15], the $k-\varepsilon$ model, proposed by Launder and Spalding [16], is investigated. This model introduces two new equations, one for the turbulent kinetic energy (k) and the other for turbulent dissipation rate (ε). The two equations are given by:

$$\nabla \cdot (\rho_m V_m k) = \nabla \cdot \left[\left(\mu_m + \frac{\mu_{tm}}{\sigma_k} \right) \nabla k \right] + G_m - \rho_m \varepsilon \quad (14)$$

$$\nabla \cdot (\rho_m V_m \varepsilon) = \nabla \cdot \left[\left(\mu_m + \frac{\mu_{tm}}{\sigma_\varepsilon} \right) \nabla \varepsilon \right] + \frac{\varepsilon}{k} \quad (15)$$

$$(C_1 G_m - C_2 \rho_m \varepsilon)$$

Where

$$\mu_{tm} = \rho_m C_\mu \frac{k^2}{\varepsilon} \quad (16)$$

$$G_m = \mu_{tm} (\nabla V_m + (\nabla V_m)^T) \quad (17)$$

With $C_1=1.44$, $C_2=1.92$, $C_\mu=0.09$, $\sigma_k=1$, $\sigma_\varepsilon=1$.

Nanofluid mixture properties

In this paper the properties of Al_2O_3 -water nanofluid are calculated using the following expressions:

Density [17] :

$$\rho_m = \phi \rho_p + (1 - \phi) \rho_f \quad (18)$$

Specific heat capacity [18]:

$$(\rho C_p)_m = \phi (\rho C_p)_p + (1 - \phi) (\rho C_p)_f \quad (19)$$

Dynamic viscosity [19] :

$$\mu_m = \mu_f + \frac{\rho_p V_B d_p^2}{72 C \delta} \quad (20)$$

Where V_B and δ are the Brownian motion of nanoparticles and the distance between nanoparticles respectively and are calculated based on:

$$V_B = \frac{1}{d_p} \sqrt{\frac{18 k_B T}{\pi \rho_p d_p}} \quad (21)$$

$$\delta = \sqrt[3]{\frac{\pi}{6 \phi}} d_p \quad (22)$$

C in Equation 14 is defined as:

$$C = \frac{(C_1 d_p + C_2) \phi + (C_3 d_p + C_4)}{\mu_f} \quad (23)$$

Where C_1 , C_2 , C_3 and C_4 are given as:

$$\begin{aligned} C_1 &= -0.000001133, C_2 = -0.000002771 \\ C_3 &= 0.00000009, C_4 = -0.000000393 \end{aligned} \quad (24)$$

Thermal conductivity [20]:

$$\frac{k_m}{k_f} = 1 + 64.7\phi^{0.7460} \left(\frac{d_f}{d_p}\right)^{0.3690} \left(\frac{k_p}{k_f}\right)^{0.7476} \text{Pr}_f^{0.9955} \text{Re}_f^{1.2321} \quad (25)$$

Where Re_f and Pr_f can be expressed as:

$$Re_f = \frac{\rho k_B T}{3\pi\eta^2 \lambda_f} \quad (26)$$

$$Pr_f = \frac{\eta}{\rho_f \alpha_f} \quad (27)$$

Where λ_f is the MFP (mean free path) of water molecular ($\lambda_f = 0.17$ nm), k_B is Boltzmann constant ($k_B = 1.3807 \times 10^{-23}$ J/K) and η can be defined by the following equation:

$$\eta = A.10^{\frac{B}{T-C}}, A = 2.414 \times 10^{-5}, B = 247.8, C = 140 \quad (28)$$

Thermal expansion coefficient [21]:

$$\beta_m = \left[\frac{1}{1 + \frac{(1-\phi)\rho_f}{\phi\rho_p}} \frac{\beta_p}{\beta_f} + \frac{1}{1 + \frac{\phi}{1-\phi} \frac{\rho_p}{\rho_f}} \right] \beta_f \quad (29)$$

Boundary conditions

To numerically model the nanofluid flow in circular tubes fitted with diamond-shaped turbulators, the problem is solved as an axisymmetric problem. The tube's central line is modeled as the axis and the tube walls are modeled with constant heat flux boundary condition.

At the inlets of tubes, the input flow velocity, temperature and the volume fraction of nanoparticles are specified and at the outlets, the constant pressure boundary condition is applied.

Numerical methods

The numerical simulation is performed using the finite volume method. A second order upwind method is used for the convective and diffusive terms and the SIMPLE algorithm is employed to solve the coupling between the velocity and pressure fields. To make sure that the obtained results are independent of the size and the number of generated grids, several grids with different sizes along different directions has been tested for each tube; and it has been attempted to consider for each tube the best grid, with the highest accuracy and the lowest computation cost. The examined tubes contain between 2 and 3 million elements.

Figure 2 shows a sample of grid generation for a tube containing diamond-shaped turbulators.

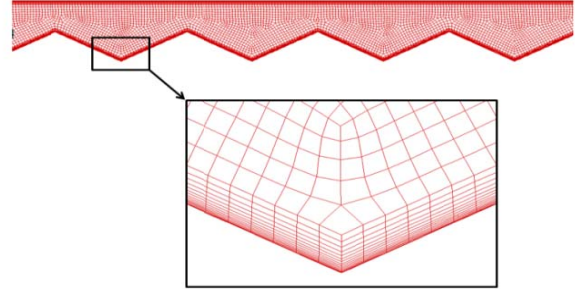


Fig. 2. A sample of grid generation

Validations

To attain the confidence about the simulations, it is necessary to compare the results with the available related data. Figures 3 and 4 compare the local heat transfer coefficient (h) in laminar and turbulent flows respectively in a plain tube of present study with the available data of Kim et al. [22] and Ebrahimi et al. [23] (laminar flow) and Bianco et al. [24] (turbulent flow). As is evident from these figures, the present simulations agree well with the available data.

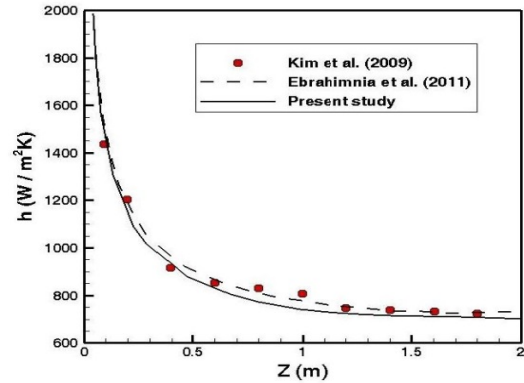


Fig.3. Comparison of local heat transfer coefficient of the present numerical simulation with the available data for laminar flow in a plain tube, $Re=1460$, $q'' = 2089 \text{ W/m}^2$, $d_p=20 \text{ nm}$, $\Phi=3\%$

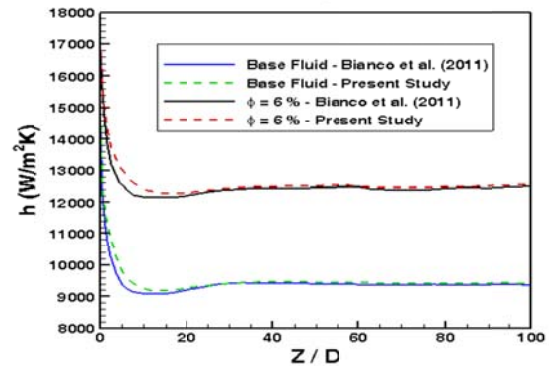


Fig. 4. Comparison of local heat transfer coefficient of the present numerical simulation with the available data for turbulent flow in a plain tube, $Re=20000$, $q'' = 500 \text{ kW/m}^2$, $d_p=38 \text{ nm}$

RESULTS

In this paper, the Al₂O₃-water nanofluid flow in laminar and turbulent flows inside tubes fitted with diamond-shaped turbulators is numerically modeled. In this section, the effects of different geometrical and non-geometrical parameters on the nanofluid flow will be explored for laminar and turbulent flows in tubes. In all of the investigations, the Reynolds numbers for the laminar and turbulent flows are 1000 and 10000, respectively. Moreover in all of the investigations related to turbulent flows, parameter γ^+ is controlled and it has appropriate values.

Effect of turbulator geometry

In this paper, the effects of two geometrical parameters of turbulators (TR and θ) on nanofluid flow field have been evaluated. Since in the flow within tubes, in addition to the amount of heat transfer, the amount of nanofluid pressure drop and thus the amount of shear stress at tube walls is also important, the latter parameter has also been investigated. Since by placing the turbulators in the way of fluid flow the shear stress of walls undergoes severe fluctuations, these shear stresses cannot be considered locally in a figure so their average values is presented in a table format. Figure 5 shows the effect of TR variations on the local heat transfer coefficient.

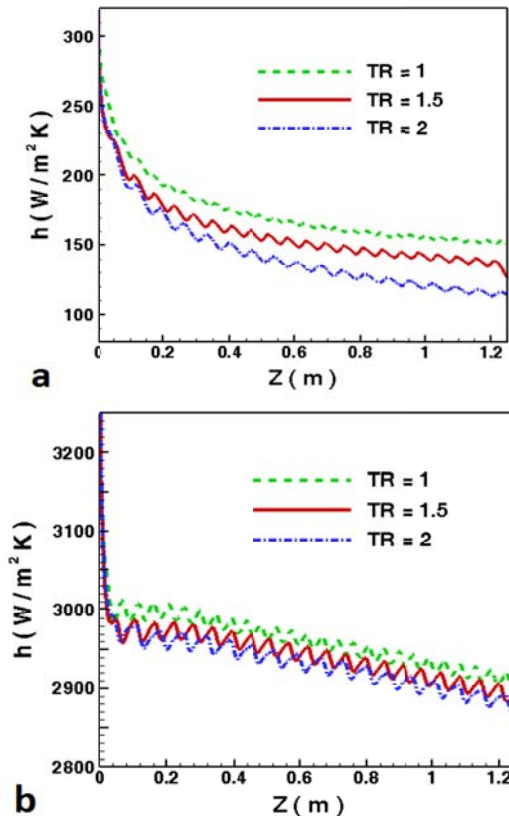


Fig. 5. Effects of tail length ratio on local heat transfer coefficient, $\theta=30^\circ$, $q'' = 4000 W/m^2$, $d_p=40 nm$, $\Phi=1.5\%$, (a) laminar $Re=1000$ (b) turbulent $Re=10000$

Also, the impact of this parameter on the shear stress of tube walls has been presented in Table 1.

Table 1
Effect of different parameters on the average wall shear stress.

Effect of Variable	Value	τ (pa)		Changes in τ (%)	
		Laminar	Turbulent	Laminar	Turbulent
TR	1	0.047	1.23	0.00	0.00
	1.5	0.041	1.15	-12.76	-6.50
	2	0.033	0.98	-29.78	-20.32
θ (deg)	15	0.031	0.87	0.00	0.00
	22.5	0.037	1.03	19.35	18.39
	30	0.041	1.15	32.25	32.18
Φ (%)	0	0.040	1.13	0.00	0.00
	1.5	0.041	1.15	2.50	1.76
	3	0.043	1.19	7.50	5.30
d_p (nm)	20	0.41	1.14	0.00	0.00
	40	0.041	1.15	0.00	0.87

According to Figure 5, the heat transfer coefficient increases with the reduction of TR in both the laminar and turbulent flows. The saw tooth features in the local heat transfer coefficient diagram are due to the existence of diamond-shaped turbulators in the tube. Table 1 indicates that the reduction of TR , in addition to increasing the heat transfer coefficient, also increases the shear stress at the walls, which constitutes a disadvantage. As is observed, with the reduction of TR from 2 to 1, the wall shear stresses in laminar and turbulent flows increase by 29% and 20%, respectively. The contours of turbulent kinetic energy and the stream lines in turbulent flow have been illustrated in Figure 6 for different TR values. As is indicated in this figure, with the reduction of TR , the secondary flows and the vortices existing in flow and also the amount of turbulent kinetic energy increase. By simultaneously considering the results of these two figures, it can be concluded that the reduction of TR ultimately causes the heat transfer coefficient to increase by increasing the secondary flows, vortices and mixing of flow.

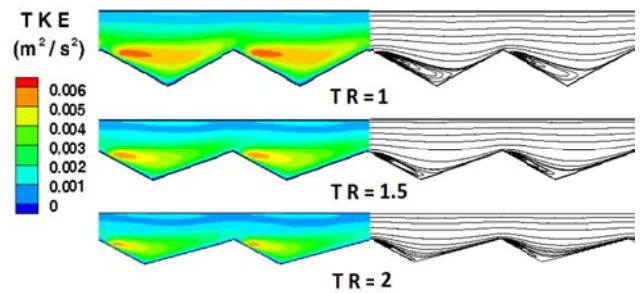


Fig. 6. Effects of tail length ratio on turbulent kinetic energy (k) and stream lines

Similarly, the effect of θ on the local heat transfer coefficient has been depicted in Figure 7. As is observed, by increasing the value of θ in both laminar and turbulent flows, the heat transfer coefficient can be increased. According to Table 1, the increase of θ also leads to the

increase of shear stress at the walls. With the increase of θ from 15° to 30° , the amount of wall shear stress in both the laminar and turbulent flows increases by about 32%.

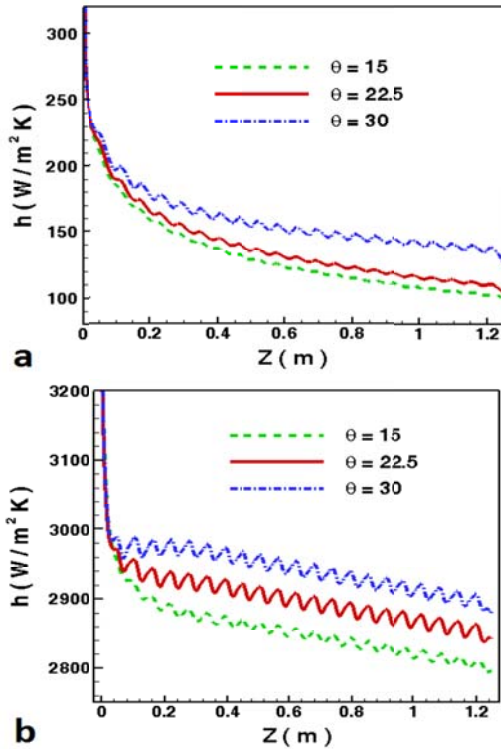


Fig. 7. Effects of vertex angle on local heat transfer coefficient, $TR=1.5$, $q'' = 4000 \text{ W/m}^2$, $d_p=40 \text{ nm}$, $\Phi=1.5\%$, (a) laminar $Re=1000$ (b) turbulent $Re=10000$

The contours of turbulent kinetic energy and the stream lines in a turbulent flow are illustrated in Figure 8 for different θ values.

According to this figure, with the increase of θ , the secondary flows and the vortices existing in flow and also the amount of turbulent kinetic energy increase. Thus, it can be concluded that the increase of θ ultimately causes the heat transfer coefficient to increase by increasing the existing secondary flows and vortices in flow.

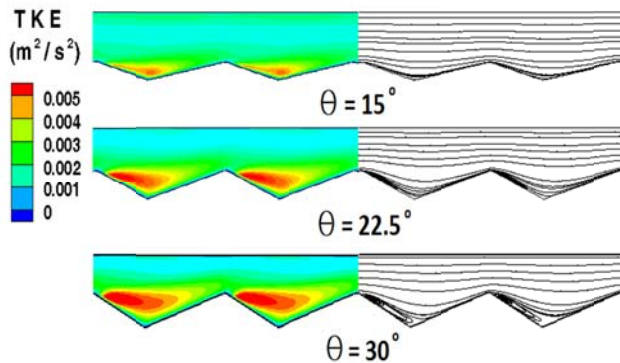


Fig. 8. Effects of vertex angle on turbulent kinetic energy (k) and stream lines

Effect of nanoparticle parameters

In this section, the effects of volume fraction and nanoparticle diameter on fluid flow in tubes fitted with diamond-shaped turbulators are investigated. Figure 9 illustrates the effect of volume fraction (as it changes from 0-3%) on the local heat transfer coefficient in laminar as well as turbulent flows. As is observed, with the increase of volume fraction from 0% to 3%, the local heat transfer coefficient increases rather significantly. Also, the third column in Table 1 indicates that with this increase in the volume fraction of nanoparticles, the amount of wall shear stress in laminar and turbulent flows increases by about 7.5 and 5.3%, respectively.

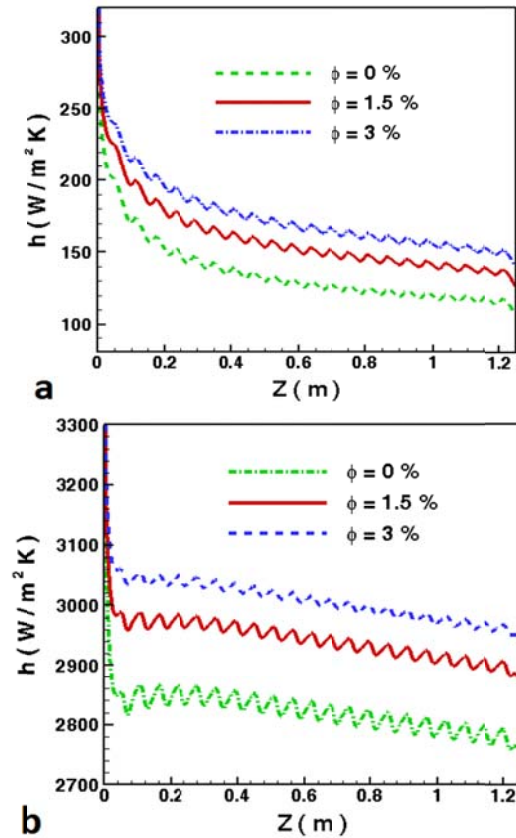


Fig. 9. Effects of nanoparticles volume fraction on local heat transfer coefficient, $TR=1.5$, $\theta=30^\circ$, $q'' = 4000 \text{ W/m}^2$, $d_p=40 \text{ nm}$, (a) laminar $Re=1000$ (b) turbulent $Re=10000$

Figure 10 shows the effect of nanoparticle diameter (as it changes from 20-40 nm) on the local heat transfer coefficient in laminar as well as turbulent flows. As is observed, with the reduction of nanoparticle diameter from 100 to 20 nm, the local heat transfer coefficient increases rather considerably.

Also, the fourth column in Table 1 indicates that with the change of nanoparticle diameter, the amount of wall shear stress increases very slightly (2.43% in laminar flows and 1.75% in turbulent flows).

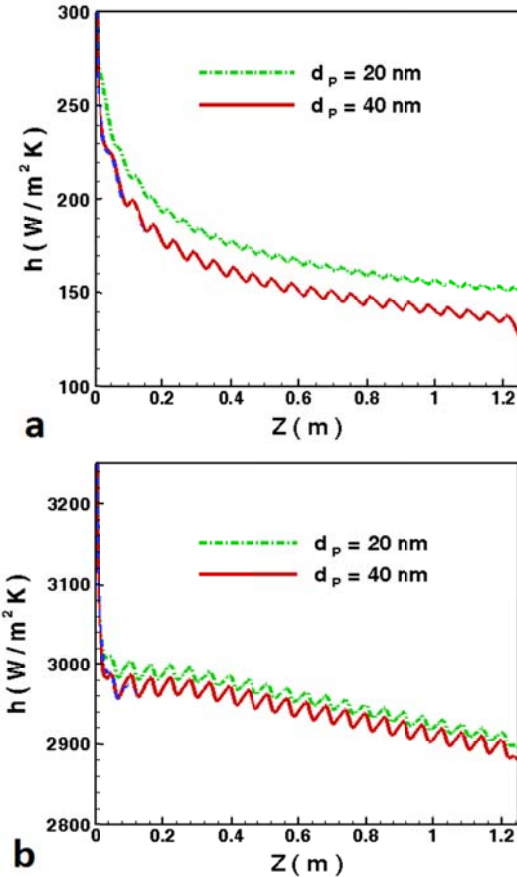


Fig. 10. Effects of nanoparticles diameter on local heat transfer coefficient, $TR=1.5$, $\theta=30^\circ$, $q'' = 4000$ W/m², $\Phi=1.5\%$, (a) laminar $Re=1000$ (b) turbulent $Re=10000$

Variations of different parameters on diamond-shaped turbulators wall Knowing how the different parameters vary on a turbulator wall can be useful in thermally designing of turbulators. The amounts of local shear stress at the wall of turbulator in laminar as well as turbulent flows have been shown in Figure 11.

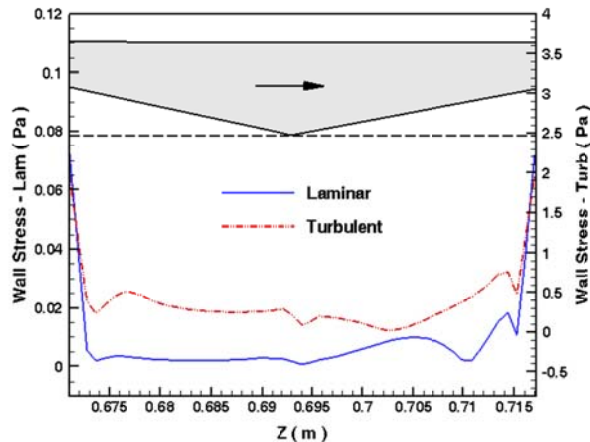


Fig. 11. Wall stress distribution across one diamond-shaped turbulators

According to this figure, the amounts of wall shear at the bulges (peaks) of the turbulators display severe changes, which cause this type of turbulators to have a considerable pressure loss. The details of the fluctuations of turbulent kinetic energy (k) and turbulent dissipation rate (ϵ) on turbulator wall have been shown in Figure 12. As is observed, the changes of both parameters are qualitatively similar and the maximum and minimum amounts of these parameters respectively appear at the peaks (bulges) and valleys (dents).

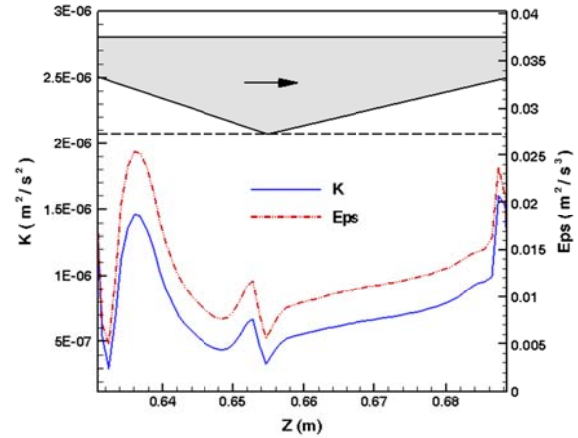


Fig. 12. Turbulent kinetic energy (k) and turbulent dissipation rate (ϵ) distributions across one diamond-shaped turbulators.

The changes of pressure over turbulator wall in laminar and turbulent flow regimes have been depicted in Figure 13. According to this figure, nanofluid has the least amount of pressure at the peaks.

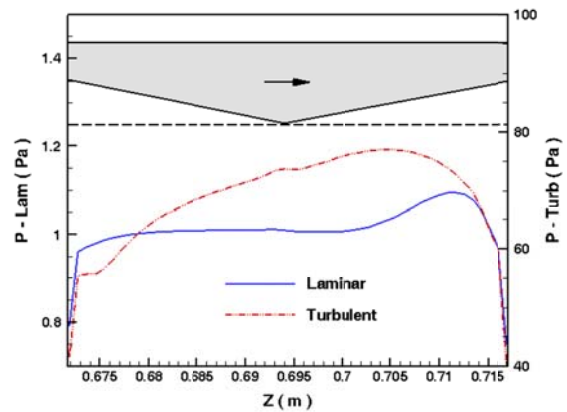


Fig. 13. Pressure distribution across one diamond-shaped turbulators

CONCLUSION

In this paper, the flow field of Al₂O₃-water nanofluid in laminar and turbulent flows inside tubes containing diamond-shaped turbulators was numerically modeled. The modeling of nanofluid was performed by employing the two-phase mixture method and applying the constant heat flux boundary condition at tube walls. The effects of

different parameters such as the geometry of turbulators (TR and θ), volume fraction and diameter of nanoparticles, etc. on the flow field of nanofluid in the tubes were investigated. It was observed that, with the reduction of TR and increase of θ , the amounts of heat transfer coefficient as well as the wall shear stress increase. Similarly, with the reduction of TR and increase of θ , the number of secondary flows, vortices and turbulent kinetic energy in the flow increase. Also, the increase in the volume fraction of nanoparticles and the reduction of the nanoparticles diameter resulted in the increase of the heat transfer coefficient and wall shear stress. Moreover, the changes of shear stress, k and ε on turbulators were investigated and it was observed that the mentioned parameters have their highest values at the hills and their lowest values at the valleys of turbulators wall.

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